More on divergences in brane world models

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Abstract

In this note a model in a space-time with compact extra dimension, describing five-dimensional fermion fields interacting with electromagnetic field localized on a brane, is presented. This model can be considered as a toy model for examining possible consequences of localization of gauge fields on a brane. It is shown that in the limit of infinite extra dimension the lowest order amplitudes of some processes in the resulting four-dimensional effective theory are divergent. Such a "localization catastrophe" can be inherent to more realistic bane world models with infinite extra dimension.

In [1] an attempt was made to construct a model with infinite extra dimension, describing spinor electrodynamics localized on a domain wall. It was found that due to the existence of non-localized fermion modes, though with large four-dimensional masses, the renormalized amplitude of the standard process of quantum electrodynamics, – the light by light scattering, is divergent. This divergence appears to be a "physical" one, i.e. it can not be removed by means of the standard renormalization procedure. It simply reflects the fact that an infinite number of fermions, as seen from the four-dimensional point of view, each with the same coupling to the vector field, contribute to the amplitude.

It is reasonable to suppose that such divergences can arise in other models with infinite extra dimension(s), where the zero mode of the gauge field is supposed to be localized on a brane. To show the origin of the problem in a simple way we will consider a toy model which possesses the same property as the one considered in [1]. It will be shown that a theory with a non-local interaction between fermions and a gauge field can result in the same effects as an initially local theory with a gauge field localized on a brane.

Let us take a model in a five-dimensional space-time with the coordinates $x^M = \{x^\mu, z\}$, $M = 0, 1, 2, 3, 4$, describing fermions interacting with an Abelian gauge field. The compact extra dimension with the coordinate $-L \le z \le L$ is supposed to form an orbifold with the points $-z$ and z identified. In what follows we will use the notation x for the coordinates x^μ. The brane is supposed to be located at a fixed point of the orbifold, say, at the point $z = 0$.

The total action of the model consists of two terms. The first term has the form

$$S_b = \int d^4x \left(-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + i\bar{\psi}_b \gamma^\mu \left(\partial_\mu - ie_b a_\mu \right) \psi_b - m\bar{\psi}_b \psi_b \right). \tag{1}$$

It describes the fermion field ψ_b living on the brane, which is coupled to the vector field a_μ with the coupling constant e_b. In our scenario a particular mechanism of localization is not taken into account. The second term includes fermions living in the whole five-dimensional space-time. In order to have a non-zero mass term for the zero Kaluza-Klein fermion mode, i.e. to have a non-zero mass gap between the localized fermion and the lowest Kaluza-Klein mode in the effective four-dimensional action, we introduce two five-dimensional spinor fields (see, for example, [2, 3]), possessing the orbifold symmetry conditions

$$\Psi_1(x, -z) = \gamma^5 \Psi_1(x, z), \tag{2}$$

$$\Psi_2(x, -z) = -\gamma^5 \Psi_2(x, z). \tag{3}$$

Thus, the second term of the total action has the form:

$$S_f = \int d^4x \int\limits_{-L}^{L} dz \left[i\bar{\Psi}_1\Gamma^N \left(\partial_N - ieA_N\right)\Psi_1 + i\bar{\Psi}_2\Gamma^N \left(\partial_N - ieA_N\right)\Psi_2 - M\left(\bar{\Psi}_1\Psi_2 + \bar{\Psi}_2\Psi_1\right)\right], \quad (4)$$

where $N = 0, .., 4$, $\Gamma^\mu = \gamma^\mu$, $\Gamma^4 = i\gamma^5$, $A_4 \equiv 0$, $A_\mu(x, z) = a_\mu(x)$.

If the five-dimensional coupling constant $e = 0$, then the effective four-dimensional theory is just the standard four-dimensional electrodynamics and we do not expect any new effects caused by the Kaluza-Klein modes. We will proceed with the possibility $e \neq 0$, which is much more interesting. We think that the most natural choice for the coupling constant e_b is $e_b = e$.

The four-dimensional effective action, coming from (4), has the form (see the detailed derivation in Appendix)

$$\begin{aligned} S_{f\_eff} = \int d^4x \Big[& i\bar{\psi}\gamma^\mu \left(\partial_\mu - iea_\mu\right)\psi - M\bar{\psi}\psi + \\ & + \sum_{n=1}^{\infty}\sum_{i=1}^{2} \left(i\bar{\psi}_i^n\gamma^\mu \left(\partial_\mu - iea_\mu\right)\psi_i^n - \mu_n\bar{\psi}_i^n\psi_i^n\right)\Big] \end{aligned} \quad (5)$$

with

$$\mu_n = \sqrt{\frac{\pi^2 n^2}{L^2} + M^2}. \quad (6)$$

Now the total four-dimensional effective action, coming from (1) and (4), has the form

$$\begin{aligned} S_{eff} = \int d^4x \Big[& -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + i\bar{\psi}_b\gamma^\mu \left(\partial_\mu - iea_\mu\right)\psi_b - m\bar{\psi}_b\psi_b + \\ & + i\bar{\psi}\gamma^\mu \left(\partial_\mu - iea_\mu\right)\psi - M\bar{\psi}\psi + \sum_{n=1}^{\infty}\sum_{i=1}^{2} \left(i\bar{\psi}_i^n\gamma^\mu \left(\partial_\mu - iea_\mu\right)\psi_i^n - \mu_n\bar{\psi}_i^n\psi_i^n\right)\Big] \end{aligned} \quad (7)$$

Several remarks are in order. First, the interaction between the five-dimensional fermions and the gauge field in (4) looks rather strange, because the vector field in the five-dimensional part of the action does not depend on the coordinate of the extra dimension, i.e. the theory appears to be non-local. Meanwhile, formally such a form of the interaction can not be discarded: the total action is invariant under the four-dimensional $U(1)$ gauge group and under the four-dimensional Lorentz transformations. The reason to choose such a form of the interaction is the following. Since we suppose that in general the field a_μ originates from some five-dimensional field A_M and represents its lowest massless mode, the most natural choice for the wave function is a constant. In particular, a motivation to have a constant wave function of the zero vector Kaluza-Klein mode is the universality of charge (see a detailed discussion of this problem in [4]). Another reason is that some mechanisms of localization of boson fields, for example, those presented in [5], indeed lead to a constant wave function of the zero localized mode of a boson field. Exactly the same situation is realized in the model presented in [1], where the mechanism of localization of the vector field (based on the mechanisms of [5]) also leads to a constant wave function of the lowest localized (massless) mode and thus to the interaction of form (4) of the five-dimensional fermions with this mode. The effective four-dimensional action of the model presented in [1] is very similar to (7), the only substantial difference being that in [1] the extra dimension is infinite. Thus, the action of form (7) can come from a more general five-dimensional local theory and it can be considered as a toy model, where a particular

mechanism of localization is not taken into account, but the localized theory (described by ψ_b and a_μ) "remembers" the fact that it originates from a more general five-dimensional theory through the non-local interaction of the vector field with the Kaluza-Klein modes described by (5). We also suppose that $M \gg m$, i.e. the mass gap between the brane localized theory and the Kaluza-Klein modes is very large. The parameter M can be considered as the energy scale at which five-dimensional effects may come into play.

Second, in a more general theory there can be Kaluza-Klein modes of the vector field, but they will be irrelevant for our analysis and we just drop them in the effective four-dimensional action. This is a reasonable assumption – for example, in the model proposed in [1] the electromagnetic current of the lowest fermion mode couples only to the massless four-dimensional mode of the vector field.

Now we are ready to consider particular effects following from action (7). As in [1], we will be interested in $\gamma\gamma \to \gamma\gamma$ scattering, where γ stands for the particle corresponding to the vector field a_μ (say, the photon). According to action (7), the corresponding amplitude in the lowest order in the coupling constant is schematically represented in Figure 1. For $\omega \ll m$, where ω

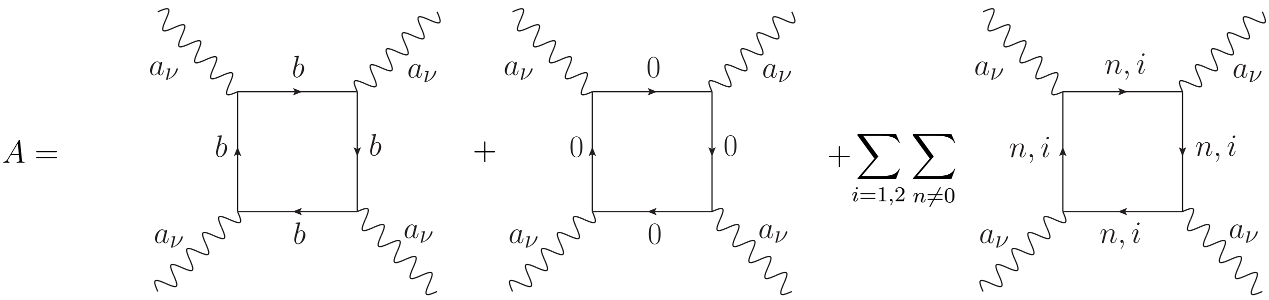

Figure 1: Diagrams corresponding to the $\gamma\gamma$ scattering amplitude. The number marking the fermion line corresponds to the number of the massive fermion Kaluza-Klein mode which is represented by this line, letter b stands for a brane localized fermion.

is the energy of the photon in the c.m. frame, the renormalized amplitude in the leading order in $\frac{\omega}{m}$ is given by [6]

$$A = \left(\frac{e^4\omega^4}{m^4} + \frac{e^4\omega^4}{M^4} + 2\sum_{n=1}^{\infty} \frac{e^4\omega^4}{\mu_n^4} \right) F(\theta), \tag{8}$$

where the function $F(\theta)$ depends on the scattering angle θ and the polarizations of the photons. We can estimate the series in A using the standard Maclaurin-Cauchy integral test for convergence:

$$\frac{L}{4M^3} - \frac{L \arctan\left(\frac{\pi}{ML}\right)}{2\pi M^3} - \frac{L^2}{2M^2(\pi^2 + M^2L^2)} = \int_1^\infty dx \frac{L^4}{(\pi^2x^2 + M^2L^2)^2} < \tag{9}$$
$$< \sum_{n=1}^{\infty} \frac{1}{\mu_n^4} = \sum_{n=1}^{\infty} \frac{L^4}{(\pi^2n^2 + M^2L^2)^2} < \int_0^\infty dx \frac{L^4}{(\pi^2x^2 + M^2L^2)^2} = \frac{L}{4M^3}.$$

Thus, amplitude (8) can be estimated as

$$A > e^4\omega^4 F(\theta) \left(\frac{1}{m^4} + \frac{1}{M^4} + \frac{L}{2M^3} - \frac{L \arctan\left(\frac{\pi}{ML}\right)}{\pi M^3} - \frac{L^2}{M^2(\pi^2 + M^2L^2)} \right). \tag{10}$$

One can see that in the limit $L \to \infty$, i.e when we pass to an infinite extra dimension, the amplitude $A \to \infty$. Note that though the mass gaps between the Kaluza-Klein modes tend to zero in this limit, the mass gap $\Delta m = M - m$ between the brane localized fermion and the lowest Kaluza-Klein mode remains intact and it can be very large. A divergence of exactly the same type arises in the model discussed in [1]. This "localization catastrophe" is the consequence of the fact that, though the vector field is localized on the brane, it interacts with the five-dimensional fermions everywhere in the bulk. Such a non-locality of the interaction leads to a pathology, and it can arise in more realistic scenarios of gauge fields localization on the brane.

It should be noted that the use of a finite cut-off scale does not solves the problem. Indeed, let us take a cut-off scale $\tilde{M}$ such that we do not take into account Kaluza-Klein modes with the masses $\mu_n > \tilde{M}$. The number of the heaviest Kaluza-Klein mode, which contribute to the amplitude, can be defined as $N = \lfloor \frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} \rfloor$, where $\lfloor \cdot \rfloor$ stands for the floor function. Thus,

$$\sum_{n=1}^{N} \frac{1}{\mu_n^4} > \sum_{n=1}^{N} \frac{1}{\tilde{M}^4} = N\frac{1}{\tilde{M}^4} > \left(\frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} - 1\right)\frac{1}{\tilde{M}^4}, \tag{11}$$

which tends to infinity in the limit $L \to \infty$.

One can expect that the effects, analogous to the one which was demonstrated above, can arise in other processes such that the corresponding Feynaman diagrams contain the field a_μ in external lines and the fermion fields in internal lines. For example, let us take the polarization operator of the field a_μ. The corresponding diagrams are presented in Figure 2.

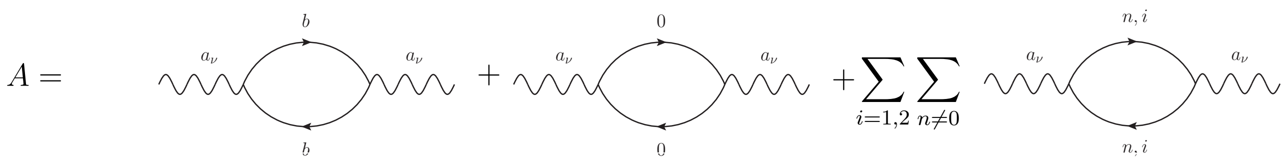

Figure 2: Diagrams which contribute to the polarization operator of the field a_ν in the leading order. As in Figure 1, the number marking the fermion line corresponds to the number of the massive fermion Kaluza-Klein mode which is represented by this line, letter b stands for a brane localized fermion.

The polarization operator has the form

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - \eta_{\mu\nu} q^2)\left(\Pi(q^2, m_b) + \Pi(q^2, \mu_0) + 2\sum_{n=1}^{\infty} \Pi(q^2, \mu_n)\right). \tag{12}$$

It is well known that for $q^2 < m_b^2$ the function $\Pi(q^2, m_b)$ can be expanded as [7]

$$\Pi(q^2, m_b) = \Pi(0, m_b) + \left.\frac{\partial \Pi(q^2, m_b)}{\partial q^2}\right|_{q^2=0} q^2 + \cdots, \tag{13}$$

where $\frac{\partial \Pi(q^2, m_b)}{\partial q^2}\Big|_{q^2=0} = \frac{16 i \pi^2}{60 m_b^2}$ and $\Pi(0, m_b)$ is logarithmically divergent. An analogous expansion can be made for the contributions of the other Kaluza-Klein fermions. Thus, in the leading order in $q^2/(\text{mass of the mode})^2$ the renormalized contribution to the polarization operator is proportional to

$$q^2\left(\frac{1}{m_b^2} + \frac{1}{\mu_0^2} + 2\sum_{n=1}^{\infty} \frac{1}{\mu_n^2}\right). \tag{14}$$

In the limit $L \to \infty$ the sum (14) also diverges. To show it, let us again introduce a cut-off scale $\tilde{M} > M$. In this case

$$\sum_{n=1}^{\infty} \frac{1}{\mu_n^2} > \sum_{n=1}^{\lfloor \frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} \rfloor} \frac{1}{\mu_n^2} > \sum_{n=1}^{\lfloor \frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} \rfloor} \frac{1}{\tilde{M}^2} = \frac{\lfloor \frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} \rfloor}{\tilde{M}^2} > \left(\frac{L\sqrt{\tilde{M}^2 - M^2}}{\pi} - 1 \right) \frac{1}{\tilde{M}^2}. \quad (15)$$

One can see that (15) indeed is divergent in the limit $L \to \infty$ so that the polarization operator diverges for $q^2 \neq 0$.

Just for a comparison, let us consider a rather different scenario – where the vector field can freely propagate in the bulk. In this case the action for the vector field can be chosen to be

$$S_{vect} = -\int d^4x dz \frac{\xi^2}{4} F_{MN} F^{MN}, \quad (16)$$

instead of the corresponding brane localized term in (1). Here the constant ξ with the dimension $\sqrt{[\text{mass}]}$ is introduced for convenience – the field A_M in this case has the standard dimension [mass]. From the very beginning we can impose the gauge $A_4 \equiv 0$. After imposing this gauge, we are left with the residual gauge transformations which are responsible for isolating the physical degrees of freedom of the massless four-dimensional vector field. We will be interested in the lowest mode of the vector field, so we take the ansatz $A_\mu(x,z) = \frac{1}{\xi\sqrt{2L}} a_\mu(x)$. Substituting it into action (16), integrating over the coordinate of the extra dimension and taking into account (5) and (1) we obtain

$$S = \int d^4x \left(-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + i\bar{\psi}_b \gamma^\mu \left(\partial_\mu - ie_4 a_\mu\right) \psi_b - m\bar{\psi}_b \psi_b + \right. \quad (17)$$

$$\left. + i\bar{\psi}\gamma^\mu \left(\partial_\mu - ie_4 a_\mu\right)\psi - M\bar{\psi}\psi + \sum_{n=1}^{\infty}\sum_{i=1}^{2} \left(i\bar{\psi}_i^n \gamma^\mu \left(\partial_\mu - ie_4 a_\mu\right) \psi_i^n - \mu_n \bar{\psi}_i^n \psi_i^n \right) \right),$$

where $e_4 = \frac{e}{\xi\sqrt{2L}}$.

Now let us calculate the amplitude corresponding to the process presented in Figure 1. It has the form

$$A = \left(\frac{e_4^4\omega^4}{m^4} + \frac{e_4^4\omega^4}{M^4} + 2\sum_{n=1}^{\infty} \frac{e_4^4\omega^4}{\mu_n^4} \right) F(\theta) = \frac{1}{4L^2\xi^4} \left(\frac{e^4\omega^4}{m^4} + \frac{e^4\omega^4}{M^4} + 2\sum_{n=1}^{\infty} \frac{e^4\omega^4}{\mu_n^4} \right) F(\theta). \quad (18)$$

Using (9) we can estimate the amplitude as

$$A < \frac{e^4\omega^4 F(\theta)}{4\xi^4} \left(\frac{1}{L^2} \left(\frac{1}{m^4} + \frac{1}{M^4} \right) + \frac{1}{2M^3 L} \right). \quad (19)$$

We see that now in the limit $L \to \infty$ the amplitude $A \to 0$. From the four-dimensional point of view the only difference between this case and the previous one with the brane localized gauge field is only in the value of the coupling constant (e versus $e/(\xi\sqrt{2L})$). From the physical point of view the difference is obvious – the second case corresponds to a local theory. Thus, in the limit $L \to \infty$ the zero mode of the gauge field A_μ ceases to be a four-dimensional particle – there is no mass gap between the zero mode and the next Kaluza-Klein mode of the vector field, as well as between the other Kaluza-Klein modes. Now all the Kaluza-Klein modes compose a single five-dimensional particle. In the resulting five-dimensional (though non-renormalizable in the usual sense) theory the corresponding amplitude, of course, is non-zero.

Conclusion

Here we considered the simplest case of Abelian gauge field. It was shown that localization of such a gauge field on a brane can result in an effective action (as an example one can consider the model discussed in [1]), analogous to those coming from multidimensional theories containing non-local interactions from the very beginning. Since one may expect pathologies in theories with non-local interactions, analogous pathologies can arise in theories with brane-localized gauge fields in more general cases. One should take this effect into account when considering brane world models with infinite extra dimensions.

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Appendix

According to the orbifold symmetry conditions (2), (3) the five-dimensional fermion fields Ψ_1 and Ψ_2 can be decomposed into Kaluza-Klein modes (see [3]) as

$$\Psi_1(x,z) = \frac{1}{\sqrt{2L}}\psi_L(x) + \frac{1}{\sqrt{L}}\sum_{n=1}^{\infty}\left(\cos\left(\frac{\pi n}{L}z\right)\psi_L^n(x) - \sin\left(\frac{\pi n}{L}z\right)\hat{\psi}_R^n(x)\right), \tag{20}$$

$$\Psi_2(x,z) = \frac{1}{\sqrt{2L}}\psi_R(x) + \frac{1}{\sqrt{L}}\sum_{n=1}^{\infty}\left(\cos\left(\frac{\pi n}{L}z\right)\psi_R^n(x) + \sin\left(\frac{\pi n}{L}z\right)\hat{\psi}_L^n(x)\right), \tag{21}$$

where $\psi_L(x) = \gamma^5\psi_L(x)$, $\psi_R(x) = -\gamma^5\psi_R(x)$, $\psi_L^n(x) = \gamma^5\psi_L^n(x)$, $\psi_R^n(x) = -\gamma^5\psi_R^n(x)$, $\hat{\psi}_L^n(x) = \gamma^5\hat{\psi}_L^n(x)$, $\hat{\psi}_R^n(x) = -\gamma^5\hat{\psi}_R^n(x)$. Substituting (20) and (21) into (4) and integrating over the coordinate z of the extra dimension, we arrive at

$$\begin{aligned} S_{f\_eff} = \int d^4x \Big[& i\bar{\psi}\gamma^\mu\left(\partial_\mu - ieA_\mu^0\right)\psi - M\bar{\psi}\psi + \sum_{n=1}^{\infty}\Big(i\bar{\psi}^n\gamma^\mu\left(\partial_\mu - ieA_\mu^0\right)\psi^n + \\ & + i\bar{\hat{\psi}}^n\gamma^\mu\left(\partial_\mu - ieA_\mu^0\right)\hat{\psi}^n - \frac{\pi n}{L}(\bar{\hat{\psi}}^n\psi^n + \bar{\psi}^n\hat{\psi}^n) - M(\bar{\psi}^n\psi^n - \bar{\hat{\psi}}^n\hat{\psi}^n)\Big)\Big] \end{aligned} \tag{22}$$

with

$$\psi(x) = \psi_L(x) + \psi_R(x), \tag{23}$$

$$\psi^n(x) = \psi_L^n(x) + \psi_R^n(x), \tag{24}$$

$$\hat{\psi}^n(x) = \hat{\psi}_L^n(x) + \hat{\psi}_R^n(x). \tag{25}$$

We see, that the mass matrix is non-diagonal. In order to bring it to a diagonal form, we use the rotation

$$\psi^n(x) = \psi_1^n(x)\cos(\theta_n) + \psi_2^n(x)\sin(\theta_n), \tag{26}$$

$$\tilde{\psi}^n(x) = \psi_1^n(x)\sin(\theta_n) - \psi_2^n(x)\cos(\theta_n) \tag{27}$$

with

$$\tan(2\theta_n) = \frac{\pi n}{ML} \tag{28}$$

and obtain

$$S_{f\_eff} = \int d^4x \left[i\bar{\psi}\gamma^\mu \left(\partial_\mu - ie_4 A^0_\mu\right)\psi - M\bar{\psi}\psi + \sum_{n=1}^{\infty} \left(i\bar{\psi}^n_1 \gamma^\mu \left(\partial_\mu - ie_4 A^0_\mu\right)\psi^n_1 + \right.\right. \tag{29}$$
$$\left.\left. + i\bar{\psi}^n_2 \gamma^\mu \left(\partial_\mu - ie_4 A^0_\mu\right)\psi^n_2 - \mu_n(\bar{\psi}^n_1\psi^n_1 - \bar{\psi}^n_2\psi^n_2)\right)\right],$$

where $\mu_n = \sqrt{\frac{\pi^2 n^2}{L^2} + M^2}$. We see that the mass terms of the fields ψ^n_2 have unconventional sign. But with the help of the standard redefinition $\psi^n_2 \to \gamma^5\psi^n_2$ we can bring action (29) to the standard form (5).

It should be noted that one can find a similar doubling of the number of four-dimensional effective degrees of freedom in the case of one five-dimensional massive fermion in a space-time with one compact extra dimension, where the extra dimension does not possess the orbifold symmetry. Thus, the number of effective four-dimensional degrees of freedom in the case of two five-dimensional fermions in a space-time with a compact extra dimension forming the S^1/Z_2 orbifold and in the case of a single five-dimensional fermion in a space-time with a compact extra dimension without the orbifold symmetry is the same, i.e. the existence of the orbifold symmetry reduces almost by half the number of the Kaluza-Klein modes coming from each five-dimensional fermion.

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